IJSSR International Journal of Social Service and Research

Vol. 5, No. 6, June 2025
e-ISSN: 2807-8691 | p-ISSN: 2807-839X

Lagrange Method with Mean Semivariance Approach in Forming an Optimal Portfolio

Novita Koes Wardani, Farikhin, Titi Udjiani S.R.R.M
Universitas Diponegoro Semarang, Indonesia
*e-mail: novitakoeswardani@gmail.com, farikhin@lecturer.undip.ac.id, udjianititi@lecturer.undip.ac.id

Keywords

Portfolio Optimization, Mean-Variance, Mean-Semivariance, Downside Risk, Value at Risk (VaR), Conditional Value at Risk (CVaR)

ABSTRACT

Investment decisions involve the allocation of resources in the present with the expectation of gaining future returns, but they are inherently accompanied by unavoidable risks. These risks need to be managed through the construction of an optimal portfolio. The main issue examined in this study is the limitation of the Mean-Variance approach in addressing downside risk, which is particularly relevant under unstable market conditions. Therefore, an alternative approach, Mean-Semivariance, is integrated with the Lagrange method to obtain a more effective portfolio optimization solution. This study aims to construct a mathematical model for an optimal portfolio that explicitly accounts for downside risk. The model is formulated through an objective function and a system of constraints solved using the Lagrange multiplier method. The results indicate that the Mean-Semivariance approach yields more conservative portfolio weights compared to the Mean-Variance approach. Risk evaluation using Value at Risk (VaR) and Conditional Value at Risk (CVaR) shows that the portfolio optimized through the Mean-Semivariance approach provides better protection against extreme loss potential. Thus, this approach can be relied upon as a more responsive portfolio optimization strategy toward negative risk under volatile market dynamics.

INTRODUCTION

Investing is a complex and important decision for individuals, companies, and financial institutions. This involves committing to a certain amount of funds or other resources at this time, in the hope of making a profit or return in the future. However, every investment decision also has risks that investors must face. These risks can come from various factors such as market fluctuations, changes in economic policies, or unexpected global events. (Tandelilin, 2010).

One of the commonly used strategies for managing risk in investments is to form an investment portfolio. A portfolio is a combination of two or more assets that an investor chooses to allocate over a certain period of time, with the aim of achieving the desired level of return with acceptable risk. Establishing a good and optimal portfolio is the key to success in achieving investment goals. (Maruddani, 2019).

In the context of portfolio formation, investors always strive to minimize the risks associated with investing, while still obtaining optimal returns. In the capital market, efficient stock selection is very important. Efficient stocks are stocks that provide maximum returns at a certain level of risk or minimum risk at a certain level of return. Therefore, efficient stock selection is a key element in the process of building an effective portfolio. (Markowitz, 2009).

A commonly used approach in portfolio formation is the Mean-Variance approach, which was introduced by Harry Markowitz in 1952. This approach assumes that investors are only concerned with the level of risk (variance or standard deviation) and the expected return from their portfolio. Although this approach provides a solid foundation in modern portfolio theory, some weaknesses have been identified. One of the main drawbacks of the Mean-Variance approach is its inability to effectively handle negative risks. Negative risks, such as high price volatility or large losses, can have a significant impact on portfolio performance and investment decisions. To overcome this shortcoming, an alternative approach was developed, namely Mean Semivariance. (Tandelilin, 2010).

In addition, the integration of the Lagrange Method has been proposed as a powerful optimization tool in solving the problem of portfolio formation. By including constraint constraints in the form of Lagrange constraints, it is possible to find the optimal solution to optimization problems. Integration of the Lagrange Method with the alternative approach of Mean Variance. Like the research conducted by the State, et al. (2021) which examined the Markowtiz Mean Variance Model using the Lagrange Method, other research related to this problem was also conducted by Dai, Z., & Kang, J. (2022), Kumar, R. R., Stauvermann, P. J., & Smiths, A. (2022), Anugrahayu, M., & Azmi, U. (2023), and Muthohiroh, U (2021).

However, although there have been proposals to integrate the Lagrange Method with the Mean Variance approach, there is still a significant need for further research in terms of a deeper understanding of the effectiveness of these approaches in the formation of an optimal investment portfolio using the development of the Mean Variance, i.e. using the Mean Semivariance. Therefore, this study aims to explore the potential of integrating the Lagrange Method with the Mean Semivariance approach in forming an optimal investment portfolio.

Thus, this research is expected to make a significant contribution to the practical and theoretical understanding of the formation of an effective and efficient investment portfolio. The results of this study are expected to provide valuable guidance for investors, investment managers, and financial decision-makers in better managing their investment portfolios.

The purpose of writing this thesis is to Integrate the Lagrange Method with the Mean Semivariance approach in the formation of investment portfolios. Analyze the optimal portfolio results of the integration of the Lagrange Method with the Mean Semivariance approach in the formation of investment portfolios. Analyze Var in each of these approaches. The benefits of this research are: Adding insight and knowledge about research for authors and readers in the integration of the Lagrange Method with the Mean Semivariance approach in the formation of investment portfolios. As a scientific reference as well as for further research. As a reference for investors in making investment decisions.

METHOD

This thesis research was carried out from January to June 2025, using literature study methods and computer laboratories. Literature studies are conducted by gathering sources from relevant books, journals, and scientific papers to determine the best combination of methods in portfolio optimization. After a literature review, data processing was carried out using Microsoft Excel and Python, focusing on secondary historical closing price data on the shares of Bank Rakyat Indonesia (BBRI), Bank Central Asia (BBCA), and Bank Mandiri (BMRI) in USD, for a period of 1 year from March 13, 2024 to March 10, 2025. The research process begins with converting closing price data into return data, followed by normality tests and calculations of correlation coefficients between stocks. Then, the average return, variance, covariance, and standard deviation for each stock are calculated. The Lagrange method is used to determine the optimal portfolio weight with both the Mean-Variance and Mean-Semivariance approaches. Furthermore, portfolio performance is evaluated and return value simulation is carried out to see the behavior of the portfolio in various scenarios. The maximum loss estimate is calculated by measuring the VaR at a 95% confidence level, repeated to reflect variations in different VaR values, before finally determining the Expected Shortfall value with the appropriate formula. The planned research schedule includes all the necessary steps within six months, from literature study to thesis hearings, which are detailed in the schedule of activities table. The research flow is also presented in a diagram to provide a clear picture of the process being carried out.

RESULTS

Development of Mathematical Models and Integration of the Lagrange Method

This study uses the Lagrange Method to optimize the formation of investment portfolios by considering two risk approaches, namely Mean-Variance and Mean-Semivariance. Mean-Variance considers the overall fluctuations of an asset's returns, both upward and downward, while Mean-Semivariance focuses only on downside risk or negative fluctuations in returns. The portfolio optimization method aims to find the optimal weight of each asset in the portfolio so that a combination that provides maximum returns with minimal risk is obtained. The Lagrange method is used to ensure that optimization still meets certain constraints, such as the portfolio's target return and the total investment weight that must be worth one.

Mean-Variance Model with the Lagrange Method

The Mean-Variance approach is based on the Markowitz (1952) portfolio optimization model, which aims to minimize the total risk measured by the variance of portfolio returns. The objective function of this model is to minimize the variance of the portfolio, which is given by the equation:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x \sigma_{ij}$$

information:

x_i : proportion of funds invested in shares i

x_j : the proportion of funds invested in J shares

σ_ij: Covariance between stock returns I and J

With the following constraint/limiter functions:

Optimization is carried out with the constraint that the expected return of the portfolio must be the same as the target return r_0 which is written as:

$$\phi_1(x) = \sum_{i=1}^{n} r_i x_i = r_0, \qquad r_{min} \leq r_0 \leq r_{max}$$

information:

r_i : Expected Return of Shares I

x_i : proportion of funds invested in shares i

n : total number of shares in the portfolio

r_0: the target return that the portfolio wants to achieve

In addition, the total investment weight must be worth one, namely:

$$\phi_2(x) = \sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0, \qquad i = 1,2,\dots,n$$

information:

x_i : proportion of funds invested in shares i

n : total number of shares in the portfolio

To complete this optimization the Lagrange Method is used, by defining the auxiliary functions as follows:

$$G(x) = F(x) + \lambda_1 \phi_1(x) + \lambda_2 \phi_2(x)$$

$$\boldsymbol{G(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j\, \sigma_{ij} - \lambda_1\left(\sum_{i=1}^{n} r_i x_i - r_0\right) - \lambda_2\left(\sum_{i=1}^{n} x_i - 1\right)} \qquad \text{IV.1}$$

Since we're going to minimize the Lagrange function, we take a partial derivative of each variable x_k

$$\boldsymbol{\frac{\partial}{\partial x_k} = \sum_{j=1}^{n} x_j \sigma_{kj} + \sum_{i=1}^{n} x_i \sigma_{ik} - \lambda_1 r_k - \lambda_2 \ = 0} \qquad \text{IV.2}$$

Because then: $\sigma_{ij} = \sigma_{ji}$,

$$\frac{\partial}{\partial x_k} = 2\sum_{j=1}^{n} x_j \sigma_{kj} - \lambda_1 r_k - \lambda_2 \ = 0 \qquad \text{IV.3}$$

The results of the partial derivative are:

$$2\sum_{j=1}^{n} x_j \sigma_{kj} - \lambda_1 r_k - \lambda_2 \ = 0 \text{ to } \qquad k = 1,\dots,n \qquad \text{IV.4}$$

information:

G(x) : new destination function

F(x) : Original Purpose Function

φ_1 (x) and φ_2 (x) : constraint functions associated with variable x

r_k : Expected return of K shares

λ_1 and λ_2 : Lagrange multiplier

σ_kj : Covariance between K and J stock returns

The 1V.4 equation can be converted into the form of a matrix as follows:

Define matrix symbols

Suppose:

Suppose:

$x = [x_1, x_2, \dots, x_n]^T$ is the vector of the weight of the stock

$r = [r_1, r_2, \dots, r_n]^T$ is the expected *return* vector

$\Sigma = [\sigma_{kj}] \in \mathbb{R}^{nxn}$ is the matrix of covariance between stocks

Partial derived matrix

The result of the system derived from Lagrange becomes:

$$2\Sigma x = \lambda_1 r + \lambda_2 1 \qquad \text{IV.5}$$

$$\text{with } 1 = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$$

The constraint/limiting function of Lagrange is as follows:

$$r^T x = r_0$$
$$1^T x = 1$$

The complete matrix system combination is as follows

$$\begin{bmatrix} 2\Sigma & -r & -1 \\ r^T & 0 & 0 \\ 1^T & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ r_0 \\ 1 \end{bmatrix}$$

***Mean-Semivariance Model* with the Lagrange Method**

The *Mean-Semivariance* approach focuses more on *downside* risk, i.e. only considering negative deviations from the average return. The semivariance function is given as follows:

$$\min \frac{1}{T} \sum_{i=1}^{T} \left\{ \left[\sum_{i=1}^{n} (r_{it} - r_i) x_i \right]^- \right\}^2$$

information:

T : the number of historical time periods used for estimation

r_{it}: *return* of assets in time period ti

r_i : the average return of the asset over that period of time$i$

With the following constraint/limiter functions:

Optimization is carried out with the constraint that *the* expected return of the portfolio must be equal to the target *return* r_0 written as:

$$\phi_1(x) = \sum_{i=1}^{n} r_i x_i = r_0, \qquad r_{min} \le r_0 \le r_{max}$$

information:

r_i : Expected *return* of the stock i

x_i : the proportion of funds invested in stocks i

n : the total number of shares in the portfolio

r_0: the *target return* that the portfolio wants to achieve

In addition, the total investment weight must be worth one, namely:

$$\phi_2(x) = \sum_{i=1}^{n} x_i = 1$$

information:

x_i : the proportion of funds invested in stocks i

n : the total number of shares in the portfolio

To complete this optimization the Lagrange Method is used, by defining the auxiliary functions as follows:

$$G(x) = F(x) + \lambda_1 \phi_1(x) + \lambda_2 \phi_2(x)$$

$$\boldsymbol{G(x) = \frac{1}{T} \sum_{t=1}^{T} \left\{ \left[\sum_{i=1}^{n} (r_{it} - r_i) x_i \right] \right\}^2 - \lambda_1 \left(\sum_{i=1}^{n} r_i x_i - r_0 \right) - \lambda_2 \left(\sum_{i=1}^{n} x_i - 1 \right)} \qquad \text{IV.6}$$

To obtain the optimal solution, we will look for a partial derivative of each variable x_j

$$\frac{\partial}{\partial x_j} = \frac{2}{T}\sum_{t=1}^{T}\left(\sum_{i=1}^{n}(r_{it} - r_i)\,x_i\right)(r_{jt} - r_j) - \lambda_1 r_j - \lambda_2 \qquad \text{IV.7}$$

The results of the partial derivative are:

$$\frac{2}{T}\sum_{t=1}^{T}\left(\sum_{i=1}^{n}(r_{it} - r_i)\,x_i\right)(r_{jt} - r_j) = \lambda_1 r_j + \lambda_2 \text{ to } j = 1,2,\dots,n \qquad \text{IV.8}$$

information:

$G(x)$: New Purpose Function

$F(x)$: Original Purpose Function

$\phi_1(x)$ and $\phi_2(x)$: the constraint function associated with the variable x

λ_1 and : Lagrange multiplierλ_2

r_{it} : *return* of shares in time period ti

r_{jt} : *Return* of shares in time period tj

It can be converted into the form of a matrix as follows:

Define matrix symbols

Suppose:

$R \in \mathbb{R}^{n\text{x}T}$ is the stock *return* matrix, with $R_{i,t} = r_{it}$

$\bar{r} \in \mathbb{R}^n$ is the vector of the average return of the stock, $\bar{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{it}$

$\tilde{R}_{i,t} = r_{it} - \bar{r}_i$ is the *return* that has been reduced to the average

$\tilde{R} \in \mathbb{R}^{n\text{x}T}$ is a centered return matrix

$\sum = \frac{1}{T}\tilde{R}\tilde{R}^T$ is the matrix of covariances

$x = [x_1, x_2, \dots, x_n]^T$ is the vector of the weight of the stock

$r = [r_1, r_2, \dots, r_n]^T$ is the expected *return* vector (the average of each line R

Partial derived matrix

The result of the system derived from Lagrange becomes:

$$2\sum x = \lambda_1 r + \lambda_2 1$$

with $1 = [1, 1, \dots, 1]^T \in \mathbb{R}^n$

The constraint/limiting function of Lagrange is as follows:

$r^T x = r_0$

$1^T x = 1$

The complete matrix system combination is as follows

$$\begin{bmatrix} 2\sum & -r & -1 \\ r^T & 0 & 0 \\ 1^T & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ r_0 \\ 1 \end{bmatrix}$$

Data Collection

In this study, the data used is secondary data in the form of the daily closing price of shares from three banking issuers in Indonesia, namely Bank Rakyat Indonesia (BBRI), Bank Central Asia (BBCA), and Bank Mandiri (BMRI). This data is taken from Yahoo Finance, which is one of the most trusted sources for global financial information, including stock prices, trading volumes, and other market indicators.

The data period used in this study is March 13, 2024 to March 10, 2025. This time range is chosen to cover stock price movements within one year, so that it can provide a more representative picture of the return patterns and volatility of stocks that will be used in portfolio optimization analysis. The data is summarized in Appendix 1, for example for some of the data is shown in Table 1 below:

Table 1. Closing Data of BBRI, BBCA, and BMRI Stock Prices

Date	BBRI	BBCA	BMRI
13/03/2024	6.400	10.000	7.275
14/03/2024	6.150	10.325	7.400
15/03/2024	5.975	10.150	7.400
18/03/2024	6.000	10.150	7.175
19/03/2024	6.000	10.175	7.275
20/03/2024	6.100	10.125	7.050
21/03/2024	6.100	10.125	7.050
22/03/2024	6.125	10.100	7.050
25/03/2024	6.250	10.075	7.250
…	…	…	…
05/03/2025	3.840	9.000	4.860
06/03/2025	3.950	8.975	4.870
07/03/2025	3.810	8.925	4.840
10/03/2025	3.760	8.925	4.710

In this study, only closing price data is used, because this price reflects the final value of the transaction that has occurred in the market on that day.

Data Processing

Once the stock price data is collected and validated, the next step is to process the raw data into a form that can be further analyzed. In the context of portfolio optimization, the calculation is based on stock returns, not direct stock prices. Therefore, the first stage in data processing is to convert daily stock prices into daily returns. Daily return is a measure of the level of profit or loss of an asset from day to day, which is obtained by comparing the closing price of a stock on a given day with the closing price of the previous day.

Daily Return Calculation and Normality Test

Stock return is a measure of the change in the price of an asset from one period to the next. In this study, stock returns were calculated using logarithmic returns, which are more commonly used in financial analysis because they have additive properties over a longer period of time.

The daily return for each stock is calculated using equation II.5 as follows:

$$R_t = ln\left(\frac{P_t}{P_{t-1}}\right)$$

with as $P(t_i)$ the stock price for a t_i period and is the price of the asset in the previous period by $P(t_{i-1}) i = 1,2,3, \dots, n$.

The daily return shows the percentage change in the stock price from one day to the next, which is a key indicator in risk measurement and portfolio optimization calculations. For example, if the closing price of BBRI shares on day t is IDR 6,400, and on the previous day (t-1) it is IDR 6,150, then the daily return is calculated as follows:

$$R_t = ln\left(\frac{P_t}{P_{t-1}}\right)$$
$$R_1 = ln\left(\frac{P_1}{P_{1-1}}\right)$$
$$R_1 = ln\left(\frac{6.150}{6.400}\right)$$
$$R_1 = -0{,}0390625 \text{ or } 3{,}91\%$$

This return shows that BBRI's share price decreased by 3.91% on that day. The same calculation is also carried out for BBCA and BMRI shares every day, so that daily returns are obtained for all stocks in the dataset.

The calculation was applied to the closing price data of BBRI, BBCA, and BMRI shares using the help of a python application program to generate a daily return dataset, with syntax entered as follows:

```
import pandas as pd
import numpy as np
from scipy.stats import norm, kstest
from numpy.linalg import inv

# Membaca data
df = pd.read_excel("Harga Penutupan Saham.xlsx", sheet_name="Sheet1")

# Mengatur tanggal sebagai indeks
df.set_index("Date", inplace=True)

# Menghitung return harian
df_returns = df.pct_change().dropna()

# Output return harian
print(df_returns.head())
```

Figure 2. Syntax of Stock Return Calculation

The results of the transformation of data from stock price to stock return can be presented in the following Table 2:

Table 2. Results of Calculation of BBCA, BBCA, and BMRI Stock Returns.

t	*Return*		
	BBRI	**BBCA**	**BMRI**
1	-3,91%	3,25%	1,72%
2	-2,89%	-1,71%	0,00%
3	0,42%	0,00%	-3,09%
…	….	…	…
231	0,00%	0,57%	-1,23%
232	4,53%	1,68%	0,41%
233	2,82%	-0,28%	0,21%
234	-3,61%	-0,56%	-0,62%
235	-1,32%	0,00%	-2,72%

Each column in the table shows the daily returns of each stock in the period analyzed. The complete data of the results of the daily return calculation is summarized in Appendix 2.

One of the important assumptions in portfolio optimization analysis is that stock returns are distributed normally. The normal distribution is a key assumption in the Markowitz Mean-Variance model, as it allows the calculation of expected returns and risks with a simpler statistical approach. Therefore, before conducting further analysis, testing is needed to ensure whether the stock return data used in this study follows a normal distribution.

In this study, the Kolmogorov-Smirnov Test (K-S Test) was used to test the normality of stock return data from BBRI, BBCA, and BMRI. If the test results show that the returns are not normally distributed, then alternative approaches can be considered, such as data transformation or the use of robust statistical methods that do not rely on assumptions of normality. The Kolmogorov-Smirnov test is a non-parametric statistical method used to determine whether a sample belongs to a particular distribution. In the context of this study, this test will be used to test hypotheses:

H_(0): Normal distributed stock returns.

H_(1): Stock returns are not normally distributed.

If the resulting p-value is greater than 0.05, then there is not enough evidence to disprove H_0, which means the stock's returns can be considered normally distributed. Conversely, if the p-value is less than 0.05, then the null hypothesis is rejected and the stock return is considered not to be normally distributed.

The Normality Test in this study uses the help of a python application program with syntax entered as follows:

```
from scipy import stats

for col in df_returns.columns:
    data = df_returns[col]
    mean, std = data.mean(), data.std()
    ks_stat, p_value = stats.kstest(data, 'norm', args=(mean, std))
    print(f"Uji K-S untuk {col}:")
    print(f"  KS Statistic: {ks_stat:.4f}")
    print(f"  p-value: {p_value:.4f}\n")
```

Figure 3. Normality Test Syntax

The output of the normality test results is summarized in Table 4.3 below.

Table 3. Normality Test Results of BBRI, BBCA, and BMRI Shares

Stock	KS Statistic	*p-value*	Hypothetical Decision
BBRI	0.0575	0.4041	Unsubtracted (Normal)H_0
BBCA	0.0739	0.1465	Unsubtracted (Normal)H_0
BMRI	0.0624	0.3066	Unsubtracted (Normal)H_0

Based on the results of the K-S test, it was found that for all three stocks tested, the p-value was greater than 0.05, which means that there is not enough evidence to reject the zero hypothesis. Thus, it can be concluded that the returns of BBRI, BBCA, and BMRI shares can be considered to be normally distributed in the period analyzed.

Calculation of Statistical Parameters

After ensuring that the stock's returns are distributed normally, the next step is to calculate the key statistical parameters that will be used in the portfolio optimization analysis. These statistical parameters include average returns, variances, standard deviations, covariances, and correlations between stocks, which play an important role in understanding the risks and relationships between assets in a portfolio. Average return is used to measure the expected profit of each stock. Variance and standard deviation are used to measure the total risk of each stock. Covariance describes the relationship between the returns of two stocks, whether they are moving in the opposite direction (positive) or in the opposite direction (negative). The correlation coefficient measures how strong the relationship between stock returns is, with values ranging from -1 to 1, where -1 means a perfect negative correlation, 1 means a perfect positive correlation, and 0 means no linear relationship between stocks.

This calculation is important in portfolio optimization, as the relationships between stocks in the portfolio affect risk diversification and optimal portfolio formation. Statistical parameters are calculated using the help of a python application program with the following syntax entered:

```
mean_returns = df_returns.mean()
variance_returns = df_returns.var()
std_dev_returns = df_returns.std()

print("Return Rata-rata:\n", mean_returns)
print("\nVariansi Return:\n", variance_returns)
print("\nStandar Deviasi:\n", std_dev_returns)
```

```
cov_matrix = df_returns.cov()
corr_matrix = df_returns.corr()

print("\nMatriks Kovariansi:\n", cov_matrix)
print("\nMatriks Korelasi:\n", corr_matrix)
```

Figure 4. Statistical Parameter Calculation Syntax

The average return is calculated using equation II.2:

$$r_i = E[R_i] = \frac{1}{T}\sum_{i=1}^{n} r_{it}$$

Using the help of the python application program, the results of calculating the average return are obtained as follows:

Table 4. Expected Return Calculation Results

Stock	BBRI	BBCA	BMRI
Average *Return* Value	-0.2019%	-0.0361%	-0.1606%

These results show that in the period analyzed, all three stocks had negative return expectations, although the value of the decline was small.

The variance of the return is calculated by the equation:

$$\sigma_i^2 = v(R_i) = E[(R_i - E[R_i])^2] = E[(R_i - r_i)^2$$

Using the help of a python application program, the results of the variance return calculation are obtained as follows:

Table 5. Return Variance Calculation Results

Stock	BBRI	BBCA	BMRI
Variansi *Return*	0.000485	0.000248	0.000484

The standard deviation return is obtained by the equation:

$$\sigma = \sqrt{\sigma^2}$$

Using the help of the python application program, the results of the standard deviation return calculation are obtained as follows:

Table 6. Results of Standard Deviation Return Calculation

Stock	BBRI	BBCA	BMRI
Standard Deviation *Return*	2.20%	1.58%	2.20%

The standard deviation shows that BBRI and BMRI shares have higher volatility than BBCA, which means that BBRI and BMRI share prices are more volatile in this analysis period.

The covariance between two shares X and Y is calculated by the equation:

$$\sigma_{ij} = \frac{1}{T}\sum_{i=1}^{n}(r_{it} - r_i)(r_{jt} - r_j)$$

Using the help of the python application program, the results of the calculation of the covariance matrix between stocks were obtained as follows:

Table 7. Covariance Matrix Calculation Results

Stock	BBRI	BBCA	BMRI
BBRI	0.000485	0.000159	0.000289
BBCA	0.000159	0.000248	0.000200
BMRI	0.000289	0.000200	0.000484

A positive covariance value indicates that all three stocks tend to move in unidirection, albeit with different intensities.

The correlation coefficient between two shares of X and Y is calculated by the following equation:

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

Using the help of the python application program, the results of the calculation of the correlation matrix between stocks were obtained as follows:

Table 8. Results of the Correlation Matrix Calculation

Stock	BBRI	BBCA	BMRI
BBRI	1.000	0.457	0.596
BBCA	0.457	1.000	0.576
BMRI	0.596	0.576	1.000

From the table, it can be seen that the correlation between BBRI and BBCA is 0.457 which indicates a moderate positive correlation. BBRI and BMRI of 0.596 which indicates a strong positive correlation. BBCA and BMRI of 0.576 which indicates a strong positive correlation.

Since there is no correlation close to 1, it means that there is still an opportunity for risk diversification in the portfolio.

Portfolio Optimization with the Lagrange Method

After calculating key statistical parameters such as average return, variance, standard deviation, covariance, and correlation between stocks, the next stage is to determine the optimal weight of each stock in the portfolio using the Lagrange Method.

Portfolio optimization is carried out with two approaches, namely mean-variance and mean-semivariance. Mean-Variance optimizes the portfolio by minimizing total risk based on the variance of stock returns, while mean-semivariance optimizes the portfolio by minimizing downside risk, i.e. only considering negative return fluctuations. The Lagrange method is used to find the optimal weight of stocks in the portfolio by considering the constraints of the target return and the total investment weight.

The optimal weight of the Mean-Variance portfolio is calculated using the model that has been constructed in subchapter IV.1, with the help of a python application program with syntax entered as follows:

```
n_assets = df_returns.shape[1]
ones = np.ones(n_assets)
expected_returns = mean_returns.values
target_return = 0.001  # Target return harian (0.1%)

# Matriks sistem persamaan Lagrange
A = np.block([
    [2 * cov_matrix.values, expected_returns.reshape(-1, 1), ones.reshape(-1, 1)]
    [expected_returns.reshape(1, -1), 0, 0],
    [ones.reshape(1, -1), 0, 0]
])

B = np.append(np.zeros(n_assets), [target_return, 1])
solution = inv(A) @ B
optimal_weights_mv = solution[:n_assets]

print("Bobot Optimal Mean-Variance:\n", optimal_weights_mv)
```

Figure 5. Syntax of Calculation of Optimal Weights of Mean-Variance Portfolios

The results of the optimal weight of the Mean Variance portfolio are as follows:

Table 9. Results of the Calculation of the Optimal Weight of the Mean-Variance Portfolio

Stock	BBRI	BBCA	BMRI
Bobot Optimal (%)	43.47%	36.70%	19.83%

Based on these results, it can be seen that to achieve the target return of 0.1% per day with minimal risk, investors should allocate 43.47% of their investment in BBRI, 36.70% in BBCA, and 19.83% in BMRI.

The optimal weight of the Mean-Semivariance protofolio is calculated using the model that has been constructed in subchapter IV.1, with the help of a python application program with the following syntax inserted:

```
# Semivariance hanya menghitung deviasi negatif dari mean
downside_deviation = df_returns.copy()
mean_port = expected_returns.mean()

for col in downside_deviation.columns:
    downside_deviation[col] = np.where(df_returns[col] < mean_port,
                                       (df_returns[col] - mean_port)**2, 0)

semivariance_matrix = downside_deviation.cov()

# Sistem persamaan Lagrange
A_semi = np.block([
    [2 * semivariance_matrix.values, expected_returns.reshape(-1, 1), ones.reshap
    [expected_returns.reshape(1, -1), 0, 0],
    [ones.reshape(1, -1), 0, 0]
])

B_semi = np.append(np.zeros(n_assets), [target_return, 1])
solution_semi = inv(A_semi) @ B_semi
optimal_weights_semi = solution_semi[:n_assets]

print("Bobot Optimal Mean-Semivariance:\n", optimal_weights_semi)
```

Figure 6. Syntax of Calculation of the Optimal Weight of the Mean-Semivariance Portfolio

The results of the optimal weights of the Mean-Semivariance portfolio are as follows

Table 10. Results of the Calculation of the Optimal Weights of the Mean-Semivariance Portfolio

Stock	BBRI	BBCA	BMRI
Bobot Optimal (%)	47,40%	38,00%	14,60%

Based on these results, it can be seen that considering downside risks, the optimal portfolio should have a larger investment allocation to BBRI and BBCA, as well as a smaller allocation to BMRI compared to the Mean-Variance approach.

From the results of these two approaches, it can be seen that in the Mean-Semivariance approach, the allocation of funds is greater to BBRI and BBCA than to the Mean-Variance approach. This is due to the fact that the downside risk of BMRI is greater, so the optimization reduces its weight to minimize the impact of negative volatility.

Evaluation of Portfolio Risk and Performance

After determining the optimal weight of the portfolio using the Lagrange Method, the next step is to evaluate the risk level of the portfolio and analyze the maximum potential losses that can occur. In this study, risk evaluation was carried out with an approach called Value at Risk (VaR) and Expected Shortfall (Conditional VaR). Value at Risk (VaR) is used to measure the maximum potential loss in a given period with a certain level of confidence, while Expected Shortfall (Conditional VaR) is used to estimate the average loss if the portfolio has a loss greater than the value of the VaR. This evaluation is important because it provides additional insight into how much potential loss the optimal portfolio has been calculated beforehand, so that investors can consider whether the level of risk generated is in line with risk tolerance.

The calculation uses the help of a python application program with the following syntax inputs:

```
portfolio_return_semi = np.dot(df_returns, optimal_weights_semi)
portfolio_std_semi = np.std(portfolio_return_semi)

for alpha in confidence_levels:
    z = norm.ppf(1 - alpha)
    var = z * portfolio_std_semi
    cvar = norm.pdf(z) / (1 - alpha) * portfolio_std_semi
    print(f"\n[Mean-Semivariance] Tingkat Kepercayaan {int(alpha*100)}%:")
    print(f"  VaR: {var:.4%}")
    print(f"  CVaR: {cvar:.4%}")
```

```
portfolio_return_mv = np.dot(df_returns, optimal_weights_mv)
portfolio_std_mv = np.std(portfolio_return_mv)

confidence_levels = [0.95, 0.99]
for alpha in confidence_levels:
    z = norm.ppf(1 - alpha)
    var = z * portfolio_std_mv
    cvar = norm.pdf(z) / (1 - alpha) * portfolio_std_mv
    print(f"\n[Mean-Variance] Tingkat Kepercayaan {int(alpha*100)}%:")
    print(f"  VaR: {var:.4%}")
    print(f"  CVaR: {cvar:.4%}")
```

Figure 6. Var and CVaR calculations

The results of the calculation of Value at Risk (VaR) and Expected Shortfall (CVaR) for the optimal Mean-Variance and Semi-Variance portfolios are as follows:

Table 11. Results of Value at Risk (Var) and Expected Shortfall (CVaR) Calculations from Mean-Variance and Mean-Semivariance Portfolios

	VaR (95%)	**VaR (99%)**	**CVaR (95%)**	**CVaR (99%)**
Mean-Variance	-2,72%	-3,84%	3,41%	4,40%
Mean-Semivariance	-2.72%	-3.85%	3.41%	4.41%

The results of the calculation of the Value at Risk (VaR) of the optimal portfolio Mean-Variance show that with a confidence level of 95%, the optimal portfolio has a maximum probability of losing of 2.72% in a single trading day. In other words, there is a 5% chance that the losses will be greater than that number. Meanwhile, for a 99% confidence level, the maximum potential loss increases to 3.84%, which means there is only a 1% chance that the actual loss will exceed this limit.

Because VaR only provides the lower limit of losses in extreme scenarios, an Expected Shortfall (CVaR) calculation is carried out which takes into account the average loss when exceeding the VaR. The results of the calculation of CVaR 95% show that if the loss exceeds the 95% VaR limit, the average loss incurred is 3.41%. As for the 99% CVaR, the average loss in the worst-case scenario increases to 4.40%.

From these results, it can be concluded that CVaR is always greater than VaR, which indicates that when the market is experiencing high volatility and the portfolio is in a losing condition. The level of loss is greater than the limit predicted by the VaR. Therefore, CVaR is a more conservative measure of risk and better at describing the potential downside of an optimal portfolio.

Meanwhile, the results of the Value at Risk (VaR) calculation for the optimal Mean-Semivariance portfolio show that with a 95% confidence level, portfolios optimized with the Mean-Semivariance approach have a maximum probability of losing a maximum of 2.72% in a single trading day. This means that there is a 5% chance that the losses will be greater than that number.

For a higher confidence level, which is 99%, the maximum potential loss increases to 3.85%, which means there is only a 1% chance that the actual loss will exceed this limit.

Since VaR only provides a lower limit of losses in extreme scenarios, an Expected Shortfall (CVaR) calculation is performed to measure the average loss that occurs if the portfolio declines outside the VaR limit. The calculation results show that if the loss exceeds the 95% VaR, the average loss incurred is 3.41%. As for the 99% CVaR, the average loss in the worst-case scenario increased to 4.41%.

In general, the VaR and CVaR results for Mean-Semivariance show very similar values to Mean-Variance, but have a slight increase in downside risk values, indicating that the Mean-Semivariance approach is more conservative in managing stock price downside risk.

CONCLUSION

Based on the results of the analysis that has been carried out, it can be concluded that the mathematical model of portfolio optimization was successfully constructed through the integration of the Lagrange method with the Mean-Variance and Mean-Semivariance approaches. In the Mean-Variance approach, risk is measured based on the overall variance of the asset's return, while in the Mean-Semivariance approach, risk is focused on the negative deviation of return, i.e. downside risk. This approach is then formulated in the form of objective functions and constraint systems that are solved by the Lagrange multiplier method. The optimization results show that the Mean-Semivariance approach results in a more conservative portfolio weight, i.e. by lowering the proportion of investments in assets that have a higher downside risk. In contrast, the Mean-Variance approach results in a more proportionate weight of the portfolio, taking into account the overall total risk of each asset. The results of the analysis show that the portfolio resulting from the integration of this method has a competitive return compared to the Mean-Variance approach. In addition, lower downside risk indicates that investors are better protected from significant losses. This model is mathematically able to efficiently allocate investment weight to assets with low risk characteristics but have the potential to generate positive returns. Furthermore, the portfolio risk evaluation was carried out by calculating the Value at Risk (VaR) and Expected Shortfall (CVaR) at 95% and 99% confidence levels. The results of the evaluation showed that the portfolio optimized with the Mean-Semivariance approach had slightly higher VaR and CVaR values compared to the Mean-Variance approach. These results show that the Mean-Semivariance approach is better able to anticipate the risk of large losses, thus providing a safer solution in volatile market conditions.

REFERENCES

Anugrahayu, M., & Azmi, U. (2023). Stock Portfolio Optimization Using Mean-Variance and Mean Absolute Deviation Model Based On K-Medoids Clustering by Dynamic Time Warping. Jurnal Matematika, Statistika dan Komputasi, 20(1), 164-183.

Bi, J., Jin, H., & Meng, Q. (2018). Behavioral mean-variance portfolio selection. European Journal of Operational Research, 271(2), 644-663.

Dai, Z., & Kang, J. (2022). Some new efficient mean–variance portfolio selection models. International Journal of Finance & Economics, 27(4), 4784-4796.

Garcia, F., González-Bueno, J., Oliver, J., & Tamošiūnienė, R. (2019). A credibilistic mean-semivariance-PER portfolio selection model for Latin America. Journal of Business Economics and Management, 20(2), 225-243.

Gupta, P., Mehlawat, M. K., Inuiguchi, M., & Chandra, S. (2014). Fuzzy portfolio optimization. Studies in fuzziness and soft computing, 316.

Huang, X. (2008). Mean-semivariance models for fuzzy portfolio selection. Journal of computational and applied mathematics, 217(1), 1-8.[L1]

Huang, Z., Wei, P., & Weng, C. (2024). Tail mean-variance portfolio selection with estimation risk. Insurance: Mathematics and Economics, 116, 218-234.

Jorion, P. (2007). Value at risk: the new benchmark for managing financial risk. McGraw-Hill.

Kliber, P., & Rutkowska-Ziarko, A. (2019, July). An algorithm for construction of a portfolio with a fundamental criterion. In Proceedings of Economics and Finance Conferences (No. 8911300). International Institute of Social and Economic Sciences.

Kumar, R. R., Stauvermann, P. J., & Samitas, A. (2022). An Application of Portfolio Mean-Variance and Semi-Variance Optimization Techniques: A Case of Fiji. Journal of Risk and Financial Management, 15(5), 190.

Ma, X., Ma, S., Xia, L., & Zhao, Q. (2022). Mean-semivariance policy optimization via risk-averse reinforcement learning. Journal of Artificial Intelligence Research, 75, 569-595.

Macedo, L. L., Godinho, P., & Alves, M. J. (2017). Mean-semivariance portfolio optimization with multiobjective evolutionary algorithms and technical analysis rules. Expert Systems with Applications, 79, 33-43.

Markowitz, H. M. (Ed.). (2009). Harry Markowitz: selected works (Vol. 1). World Scientific.

Maruddani, D. A. I. (2019). Value at risk untuk pengukuran risiko investasi saham: Aplikasi dengan program r. Ponorogo: Wade Group.

Muthohiroh, U., Rahmawati, R., & Ispriyanti, D. (2021). Pendekatan Metode Markowitz Untuk Optimalisasi Portofolio Dengan Risiko Expected Shortfall (Es) Pada Saham Syariah Dilengkapi Gui Matlab. Jurnal Gaussian, 10(4), 508-517.

Negara, I. N. W., Langi, Y. A., & Manurung, T. (2021). Analisis Portofolio Saham Model Mean–Variance Markowitz Menggunakan Metode Lagrange. d'CARTESIAN: Jurnal Matematika dan Aplikasi, 9(2), 173-180.

Rockfellar, R.T., & Uryasev, S. (2000). Optimization of Conditional Value at Risk. Journal of Risk, 2(3), 21-41.

Sadeghi, S., Marjani, T., Hassani, A., & Moreno, J. (2022). Development of optimal stock portfolio selection model in the Tehran Stock Exchange by employing Markowitz mean-semivariance model. Journal of Finance Issues, 20(1), 47-71.

Salah, H. B., Gannoun, A., & Ribatet, M. (2020). Conditional mean-variance and mean-semivariance models in portfolio optimization. Journal of Statistics and Management Systems, 23(8), 1333-1356.

Schlosser, R., & Gönsch, J. (2023). Risk-averse dynamic pricing using mean-semivariance optimization. European Journal of Operational Research, 310(3), 1151-1163.

Sugiyono. (2019). Metode Penelitian Kuantitatif, Kualitatif, dan R&D. Bandung: Alfabeta.

Tandelilin, E. (2010). Portofolio dan Investasi: Teori dan aplikasi. Kanisius.

Wrede, R. C., Spiegel, M. R., Wrede, R., & Spiegel, M. R. (2010). Advanced calculus. New York: McGraw-Hill.

Zhang, M., Nan, J., & Yuan, G. (2012). The geometric portfolio optimization with semivariance in financial engineering. Systems Engineering Procedia, 3, 217-221.

Zhang, Y., Jin, Z., Wei, J., & Yin, G. (2022). Mean–variance portfolio selection with dynamic attention behavior in a hidden Markov model. Automatica, 146, 110629.